Homework 9

Due: June 20th (Saturday), 11:59 pm

- Please submit your work on Blackboard.
- You are required to submit your work as a single pdf.
- Please make sure your handwriting is clear enough to read. Thanks.
- No late work will be accepted.
- There are five randomly picked questions (2 pts for each) that will be graded.

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(1) (a) List all cosets of $\langle [16]_{24} \rangle$ in $\mathbf{Z}_{24}$.

(b) List all cosets of $\langle ([1]_3, [2]_6) \rangle$ in $\mathbf{Z}_3 \times \mathbf{Z}_6$.

(2) For each of the subgroups $\{e, a^2\}$ and $\{e, b\}$ of D_4, list all left and right cosets.

(3) Prove that if N is a normal subgroup of G, and H is any subgroup of G, then $H \cap N$ is a normal subgroup of H.

(4) Let N be a normal subgroup of index m in G. Show that $a^m \in N$ for all $a \in G$.

(5) Let N be a normal subgroup of G. Show that the order of any coset aN in G/N is a divisor of $o(a)$, when $o(a)$ is finite.

(6) Let H and K be normal subgroups of G such that $H \cap K = \{e\}$. Show that $hk = kh$ for all $h \in H$ and $k \in K$.

(7) If N and M are normal subgroups of G, prove that NM is also a normal subgroup of G. (*Note that you need to show that NM is a subgroup of G first.*)

(8) Compute the factor group $(\mathbf{Z}_6 \times \mathbf{Z}_4)/\langle ([2]_6, [2]_4) \rangle$.

(9) Show that $\mathbf{R}^\times/\langle -1 \rangle$ is isomorphic to the group of positive real numbers under multiplication.

(10) Let H and N be subgroups of a group G, and assume that N is a normal subgroup of G. Prove the following statements.

(a) N is a normal subgroup of HN.

(b) Each element of HN/N has the form hN, for some $h \in H$.

(c) Define $\phi : H \to HN/N$ by $\phi(h) = hN$, for all $h \in H$, is an onto homomorphism.

(d) $HN/N \cong H/K$, where $K = H \cap N$.