Homework 7

Due: June 8th (Monday), 11:59 pm

- Please submit your work on Blackboard.
- You are required to submit your work as a single pdf.
- Please make sure your handwriting is clear enough to read. Thanks.
- No late work will be accepted.
- There are five randomly picked questions (2 pts for each) that will be graded.

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(1) Find the orders of each of these permutations.

(a) $(123)(2435)(132)$

(b) $(136)(278)(42537)$

(2) Find the permutations that correspond to the rigid motions of a rectangle that is not a square. Do the same for the rigid motions of a rhombus (diamond) that is not a square.

(3) Let the dihedral group D_n be given by elements a of order n and b of order 2, where $ba = a^{-1}b$.

(a) Show that $ba^m = a^{-m}b$, for all $m \in \mathbf{Z}$.

(b) Show that $ba^m b = a^{-m}$, for all $m \in \mathbf{Z}$.

(4) Find the order of each element of D_6.

(5) Let $\tau = (abc)$ and let σ be any permutation. Show that $\sigma\tau\sigma^{-1} = (\sigma(a)\sigma(b)\sigma(c))$.

(6) In general, if $(12\cdots k)$ is a cycle of length k and σ is any permutation, then

$$\sigma(12\cdots k)\sigma^{-1} = (\sigma(1)\sigma(2)\cdots\sigma(k)).$$

(7) (a) In S_4, find the subgroup H generated by (123) and (23).

(b) For $\sigma = (234)$, find the corresponding subgroup $\sigma H\sigma^{-1}$.

(8) Let permutations in S_4 act on polynomials in four variables by permuting the subscripts, as in Theorem 10 in §3.6.

(a) Which permutations in S_4 leave the polynomial $(x_1 - x_2)(x_3 - x_4)$ unchanged?

(b) Which permutations in S_4 leave the polynomial $\displaystyle\prod_{1\le i<j\le 4}(x_i + x_j)$ unchanged?