Exam II

Exam Date: June 9th (Tuesday)
Exam Length: 100 minutes

- Please submit your work on Blackboard between 9 am and 9 pm.
- You are required to submit your work as a single pdf.
- Please make sure your handwriting is clear enough to read. Thanks.
- No late work will be accepted.
- Open-book and Open-notes.
- **Honors Code:** No consulting any online sources. No consulting with each other.

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(0) Write the following honors code with your full name at the end.
I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code. As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam. **Full name**

(1) [*8 pts*] True or False:

(a) Let p be a prime number. Then $\mathbf{Z}_p \times \mathbf{Z}_p \cong \mathbf{Z}_{p^2}$.

(b) $13\mathbf{Z} \cong 17\mathbf{Z}$.

(c) Every subgroup of a non-cyclic group is non-cyclic.

(d) Let σ be any permutation in S_n. Then σ^2 must be in A_n.

(2) [*8 pts*] Let $G = \{x \in \mathbf{R} \mid x > 0 \text{ and } x \neq 1\}$, and define $*$ on G by $a * b = a^{\ln b}$.

In Homework 2 (5), we have already shown that $(G, *)$ is an abelian group with the identity element e (the natural number e).

Show that the group $(G, *)$ is isomorphic to the multiplicative group $\mathbf{R}^\times$.

(3) [*6 pts*] Let G be a finite group of order 125 (i.e., $|G| = 125$) with the identity element e. Assume that G contains an element a with $a^{25} \neq e$. Prove that G is cyclic.

(4) (a) [*3 pts*] Let $\sigma = (17593)(2467)(385) \in S_9$. Find the order of σ in S_9.

(b) [*3 pts*] Let $\tau = (14376)(2589)(23)(1457) \in S_9$. Find the order of τ in S_9.

(c) [*3 pts*] Which of the permutations σ, τ are in A_9? Show work to support your answer.

(5) (a) [*3 pts*] Let G be a group and let $g \in G$ be an element of order 100. List all possible powers of g that have order 5.

(b) [*3 pts*] Let $G = \mathbf{Z}_{100}$. List all possible choice of $[k]_{100}$ such that $\langle [k]_{100} \rangle = \langle [35]_{100} \rangle$.

(c) [*4 pts*] Give the subgroup diagram of $\mathbf{Z}_{100}$.

(6) [*9 pts*] Let $D_n = \{a^k, a^k b \mid 0 \leq k < n\}$, where $a^n = e, b^2 = e$, and $ba = a^{-1}b$. Moreover, in Homework 7 (3), we have shown that $ba^m = a^{-m}b$ for all $m \in \mathbf{Z}$.

(a) [*2 pts*] Show that $(a^k b)^2 = e$ for each $0 \leq k < n$.

(b) [*4 pts*] Find the order of each element of D_{10}.

(c) [*3 pts*] Is D_{10} isomorphic to $\mathbf{Z}_4 \times \mathbf{Z}_5$? Show work to support your answer.