Homework 7

Due: Mar 23rd (Wednesday Class)

- Please make sure your handwriting is clear enough to read. Thanks.
- No late work will be accepted.

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(1) Find the orders of each of these permutations.

(a) $(123)(2435)(132)$

(b) $(136)(278)(42537)$

(2) Find the permutations that correspond to the rigid motions of a rectangle that is not a square. Do the same for the rigid motions of a rhombus (diamond) that is not a square.

(3) Let the dihedral group D_n be given by elements a of order n and b of order 2, where $ba = a^{-1}b$. Show that $ba^m = a^{-m}b$, for all $m \in \mathbf{Z}$.

(4) Find the order of each element of D_6.

(5) Let $\tau = (abc)$ and let σ be any permutation. Show that $\sigma\tau\sigma^{-1} = (\sigma(a)\sigma(b)\sigma(c))$.

(6) If $(12\cdots k)$ is a cycle of length k and σ is any permutation, then show that $\sigma(12\cdots k)\sigma^{-1} = (\sigma(1)\sigma(2)\cdots\sigma(k))$. (Hint: It is a generalization of Question (5).)

(7) (a) In S_4, find the subgroup H generated by (123) and (23).

(b) For $\sigma = (234)$, find the subgroup $\sigma H\sigma^{-1}$. (Hint: Use Question (5) or (6))

(8)* Show that S_n is isomorphic to a subgroup of A_{n+2}. (Hint: Any odd permutation in S_n composite $\tau = (n+1\ n+2)$ is again an even permutations in S_{n+2}.)

Question (8) is a bonus question. It is optional for the students who are in Math 546. However, it is required for the students who are in Math 701I.*