

Homework 4

Due: Feb 16th (Wednesday class)

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- Please make sure your handwriting is clear enough to read. Thanks.
 - No late work will be accepted.
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- (1) Find HK in \mathbf{Z}_{16}^\times , if $H = \langle [3] \rangle$ and $K = \langle [5] \rangle$.
- (2) Find the order of the element $([9]_{12}, [15]_{18})$ in the group $\mathbf{Z}_{12} \times \mathbf{Z}_{18}$.
- (3) Prove that if G_1 and G_2 are abelian groups, then the direct product $G_1 \times G_2$ is abelian.
- (4) Construct an abelian group of order 12 that is not cyclic.
- (5) Construct a group of order 12 that is not abelian.
- (6) Let G_1 and G_2 be groups, with subgroups H_1 and H_2 , respectively. Show that $\{(x_1, x_2) \mid x_1 \in H_1, x_2 \in H_2\}$ is a subgroup of the direct product $G_1 \times G_2$.
- (7) Let G_1 and G_2 be groups, and let G be the direct product $G_1 \times G_2$. Let $H = \{(x_1, x_2) \in G_1 \times G_2 \mid x_2 = e_2\}$ and let $K = \{(x_1, x_2) \in G_1 \times G_2 \mid x_1 = e_1\}$.
 - (a) Show that H and K are subgroups of G .
 - (b) Show that $HK = KH = G$.
 - (c) Show that $H \cap K = \{(e_1, e_2)\}$.
- (8) Let F be a field, and let H be the subset of $\text{GL}_2(F)$ consisting of all invertible upper triangular matrices. Show that H is a subgroup of $\text{GL}_2(F)$.