

Homework 1

Due: Jan 26th (Wednesday class)

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- Please make sure your handwriting is clear enough to read. Thanks.
 - No late work will be accepted.
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(0-1) Read §1.1 and §1.2 to make sure understand gcd, lcm and Euclidean algorithm.

(0-2) Read and understand the proof in lecture slide No. 14 (final slide) for § 2.3.

(1) Solve the following congruences.

(a) $2x \equiv 1 \pmod{9}$

(b) $20x \equiv 12 \pmod{72}$

(2) Make addition and multiplication tables for \mathbf{Z}_4 .

(3) Find the multiplicative inverses of the given elements (if possible).

(a) $[6]$ in \mathbf{Z}_{15} .

(b) $[7]$ in \mathbf{Z}_{15} .

(4) Let $(a, n) = 1$. The smallest positive integer k such that $a^k \equiv 1 \pmod{n}$ is called the **multiplicative order** of $[a]$ in \mathbf{Z}_n^\times .

Find the multiplicative orders of $[5]$ and $[7]$ in \mathbf{Z}_{16}^\times and show that their multiplicative orders both divide $\varphi(16)$.

(5) Consider the following permutations in S_7 .

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 5 & 4 & 6 & 1 & 7 \end{pmatrix} \quad \text{and} \quad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 7 & 4 & 6 & 3 \end{pmatrix}$$

(a) Write the following permutations as a product of disjoint cycles.

(i) $\sigma\tau$ (ii) $\tau\sigma$ (iii) σ^{-1} (iv) $\sigma\tau\sigma^{-1}$

(b) Write σ and τ as products of transpositions.

(6) Write

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 4 & 10 & 5 & 7 & 8 & 2 & 6 & 9 & 1 \end{pmatrix}$$

as a product of disjoint cycles and as a product of transpositions. Find its inverse, and find its order.

(7) Find the order of each of the following permutations.

Hint: First write each permutation as a product of disjoint cycles.

(a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 6 & 7 & 5 & 1 & 8 & 2 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 9 & 8 & 7 & 3 & 4 & 6 & 1 & 2 \end{pmatrix}$

(8) Let $\sigma = (2396)(73259)(17)(487) \in S_9$.

(a) Is σ an even permutation or an odd permutation?

(b) What is the order of σ in S_9 ?