Math 546/701I—Exam I

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(1) [**15 pts**] Let $S = \{x \in \mathbf{R} \mid x \neq 3\}$. Define $*$ on S by

$$a * b = 12 - 3a - 3b + ab.$$

Prove that $(S, *)$ is a group.

(2) (a) [**6 pts**] Find the cyclic subgroup of S_8 generated by the element $(135)(68)$.

(b) [**6 pts**] Find a subgroup H of S_8 that contains 15 elements.
You do not have to list all of the elements in H. Just prove it. That is,
Prove that H (the one you find) is a subgroup of order 15 in S_8.

(3) [**12 pts**] Let G be a group and the center of G is defined as

$$Z = \{x \in G \mid xg = gx \text{ for all } g \in G\}.$$

Note: We have already showed that Z is a subgroup of G *in Homework 3 (8).*
Let H be a subgroup of G. Prove that the set

$$HZ = \{hz \mid h \in H, z \in Z\}$$

is a subgroup of G.

(4) (a) [**5 pts**] What is the order of $([15]_{20}, [20]_{24})$ in $\mathbf{Z}_{20} \times \mathbf{Z}_{24}$?

(b) [**8 pts**] What is the largest order of an element in $\mathbf{Z}_{20} \times \mathbf{Z}_{24}$? And then use your answer to show that $\mathbf{Z}_{20} \times \mathbf{Z}_{24}$ is not cyclic.

(c) [**8 pts**] Let $G = \mathbf{Z}_{10}^{\times} \times \mathbf{Z}_{10}^{\times}$. Let $H = \langle(3,7)\rangle$ and $K = \langle(7,7)\rangle$. Find HK in G. *Here,* $(3,7)$ *means* $([3]_{10}, [7]_{10})$. *Use this simplified notations in your answer.*