Homework 9

Due: June 21st (Monday), 11:59 pm

- Please submit your work on Blackboard.
- You are required to submit your work as a single pdf.
- Please make sure your handwriting is clear enough to read. Thanks.
- No late work will be accepted.
- There are five randomly picked questions **(5 pts for each)** that will be graded.

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(1) (a) List all cosets of $\langle [16]_{24} \rangle$ in $\mathbf{Z}_{24}$.

(b) List all cosets of $\langle ([1]_3, [2]_6) \rangle$ in $\mathbf{Z}_3 \times \mathbf{Z}_6$.

(2) For each of the subgroups $\{e, a^2\}$ and $\{e, b\}$ of D_4, list all left and right cosets.

(3) Prove that if N is a normal subgroup of G, and H is any subgroup of G, then $H \cap N$ is a normal subgroup of H.

(4) Let N be a normal subgroup of index m in G. Show that $a^m \in N$ for all $a \in G$.

(5) Let N be a normal subgroup of G. Show that the order of any coset aN in G/N is a divisor of $o(a)$, when $o(a)$ is finite.

(6) Compute the factor group $(\mathbf{Z}_6 \times \mathbf{Z}_4)/\langle ([2]_6, [2]_4) \rangle$.

(7) Show that $\mathbf{R}^{\times}/\langle -1 \rangle$ is isomorphic to the group of positive real numbers under multiplication.

(8) If N and M are normal subgroups of G, prove that NM is also a normal subgroup of G. (*Note that you need to show that NM is a subgroup of G first.*)

Optional: This is a bonus question. (5 points)

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