## Homework 7

## Due: June 12th (Saturday), 11:59 pm

- Please submit your work on Blackboard.
- You are required to submit your work as a single pdf.
- Please make sure your handwriting is clear enough to read. Thanks.
- No late work will be accepted.
- There are five randomly picked questions (5 pts for each) that will be graded.
- (1) Find the orders of each of these permutations.
  - (a) (123)(2435)(132)
  - (b) (136)(278)(42537)
- (2) Find the permutations that correspond to the rigid motions of a rectangle that is not a square. Do the same for the rigid motions of a rhombus (diamond) that is not a square.
- (3) Let the dihedral group  $D_n$  be given by elements a of order n and b of order 2, where  $ba = a^{-1}b$ . Show that  $ba^m = a^{-m}b$ , for all  $m \in \mathbb{Z}$ .
- (4) Find the order of each element of  $D_6$ .
- (5) Let  $\tau = (abc)$  and let  $\sigma$  be any permutation. Show that  $\sigma \tau \sigma^{-1} = (\sigma(a)\sigma(b)\sigma(c))$ .
- (6) In general, if  $(12 \cdots k)$  is a cycle of length k and  $\sigma$  is any permutation, then show that

$$\sigma(12\cdots k)\sigma^{-1} = (\sigma(1)\sigma(2)\cdots\sigma(k)).$$

- (7) (a) In  $S_4$ , find the subgroup H generated by (123) and (23).
  - (b) For  $\sigma = (234)$ , find the subgroup  $\sigma H \sigma^{-1}$ . (Hint: Use (5) or (6) above)

