

Homework 5

Due: June 2nd (Wednesday), 11:59 pm

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- Please submit your work on Blackboard.
 - You are required to submit your work as a single pdf.
 - Please make sure your handwriting is clear enough to read. Thanks.
 - No late work will be accepted.
 - There are five randomly picked questions (**5 pts for each**) that will be graded.
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- (1) Show that the multiplicative group \mathbf{Z}_7^\times is isomorphic to the additive group \mathbf{Z}_6 .
- (2) Show that the multiplicative group \mathbf{Z}_8^\times is isomorphic to the group $\mathbf{Z}_2 \times \mathbf{Z}_2$.
- (3) Show that \mathbf{Z}_5^\times is not isomorphic to \mathbf{Z}_8^\times by showing that the first group has an element of order 4 but the second group does not.
- (4) Let (G, \cdot) be a group. Define a new binary operation $*$ on G by the formula
$$a * b = b \cdot a, \text{ for all } a, b \in G.$$

Show that the group $(G, *)$ is isomorphic to the group (G, \cdot) .

- (5) Find two abelian groups of order 8 that are not isomorphic.
- (6) Let G be any group, and let a be a fixed element of G . Define a function
$$\phi_a : G \rightarrow G \text{ by } \phi_a(x) = axa^{-1}, \text{ for all } x \in G.$$

Show that ϕ_a is an isomorphism.

- (7) Let G be any group. Define $\phi : G \rightarrow G$ by $\phi(x) = x^{-1}$, for all $x \in G$.
 - (a) Prove that ϕ is one-to-one and onto.
 - (b) Prove that ϕ is an isomorphism if and only if G is abelian.
- (8) Define $*$ on \mathbf{R} by $a * b = a + b - 1$, for all $a, b \in \mathbf{R}$. Show that the group $(\mathbf{R}, *)$ is isomorphic to the group $(\mathbf{R}, +)$.

Question (8) is only for the students who are in Math 701I.