Homework 2

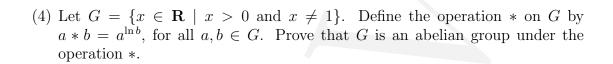
Due: May 19th (Wednesday), 11:59 pm

- Please submit your work on Blackboard.
- You are required to submit your work as a single pdf.
- Please make sure your handwriting is clear enough to read. Thanks.
- No late work will be accepted.
- There are five randomly picked questions (2 pts for each) that will be graded.
- (1) For each binary operation * defined on a set below, determine whether or not * gives a group structure on the set. If it is **not** a group, **say which axioms** fail to hold.
 - (a) Define * on \mathbf{Z} by $a*b = \max\{a, b\}$.
 - (b) Define * on \mathbf{Z} by a*b=a-b.
 - (c) Define * on \mathbf{Z} by a*b = |ab|.
 - (d) Define * on \mathbb{R}^+ by a*b=ab.
- (2) Let (G, \cdot) be a group. Define a new binary operation * on G by the formula $a*b=b\cdot a$, for all $a,b\in G$.
 - (a) Show that (G, *) is a group.

(b) Give examples to show that (G, *) may or may not be the same as (G, \cdot) .

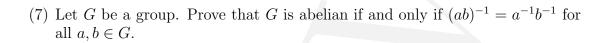
(3) Write out the multiplication table for \mathbb{Z}_7^{\times} .

¹Just note that if (iii) fails, so does (iv).



(5) Show that the set of all 2×2 matrices over \mathbf{R} of the form $\begin{bmatrix} m & b \\ 0 & 1 \end{bmatrix}$ with $m \neq 0$ forms a group under matrix multiplication. Furthermore, find all elements that commute with $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ in this group.

(6) Define * on \mathbf{R} by a*b=a+b-1, for all $a,b\in\mathbf{R}$. Show that $(\mathbf{R},*)$ is an abelian group.



- (8) Let G be a group. Prove that if $x^2 = e$ for all $x \in G$, then G is abelian.
- (9)* Show that if G is a finite group with an even number of elements, then there must exist an element $a \in G$ with $a \neq e$ such that $a^2 = e$.

Question $(9)^*$ is only for the students who are in Math 701I.