Exam II

Exam Date: June 8th (Tuesday)
Exam Length: 100 minutes

- Please submit your work on Blackboard **between 09:00 am and 11:59 pm**.
- You are required to submit your work as a single pdf.
- Please make sure your handwriting is clear enough to read. Thanks.
- You will be allowed to use your notes and the book during the exam.
- **No collaborations are allowed.**
- **No consulting any online sources is allowed.**
- **No late work will be accepted.**
- Total score: *100 points*.

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(0) [5 points] Write the following honors code.
I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code. As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam. Your name

(1) [**15 points**] True or False:

(a) $3\mathbf{Z} \cong 9\mathbf{Z}$.

(b) Let p be a prime number. Then $\mathbf{Z}_p \times \mathbf{Z}_p \cong \mathbf{Z}_{p^2}$.

(c) Every subgroup of a non-cyclic group is non-cyclic.

(d) Two finite non-cyclic groups are isomorphic if they have the same order.

(e) Let σ be any permutation in S_n. Then σ^2 must be in A_n.

(2) [**20 points**] Let $G = \{x \in \mathbf{R} \mid x > 0 \text{ and } x \neq 1\}$, and define $*$ on G by $a * b = a^{\ln b}$ for all $a, b \in G$. In Homework 2 (4), we have already shown that $(G, *)$ is an abelian group and the identity element is the natural number e. Prove that the group $(G, *)$ is isomorphic to the group $\mathbf{R}^\times$ under the standard multiplication.

(3) (a) [**8 points**] Let G be a group and let $g \in G$ be an element of order 100. List all possible powers of g that have order 5.

(b) [**8 points**] Let $G = \mathbf{Z}_{100}$. List all possible choice of $[k]_{100}$ such that $\langle [k]_{100} \rangle = \langle [15]_{100} \rangle$.

(c) [**8 points**] Give the subgroup diagram of $\mathbf{Z}_{100}$.

(4) [**24 points**] Let $D_n = \{a^k, a^k b \mid 0 \leq k < n\}$, where $a^n = e, b^2 = e$, and $ba = a^{-1}b$. Moreover, in Homework 7 (3), we have shown that $ba^m = a^{-m}b$ for all $m \in \mathbf{Z}$.

(a) [**6 points**] Show that $(a^k b)^2 = e$ for each $0 \leq k < n$.

(b) [**12 points**] Find the order of each element of D_{10}.

(c) [**6 points**] Is D_{10} isomorphic to $\mathbf{Z}_4 \times \mathbf{Z}_5$? Show work to support your answer.

(5) [**12 points**] Let G be a finite group of order 125 with the identity element e. Assume that G contains an element a with $a^{25} \neq e$. Prove that G is cyclic.