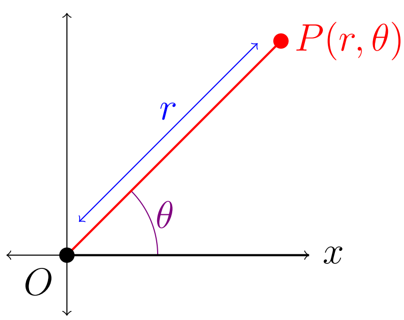
§ 11.3: Polar Coordinates

Definition: To define polar coordinates, we first fix an **origin** O (called the **pole**) and an **initial ray** from O (usually the positive x-axis). Then each point P can be located by assigning to it a **polar coordinate pair** (r, θ) in which r gives the directed distance from O to P and θ gives the directed angle from the initial ray to the ray OP.



Just like trigonometry, θ is positive when measured anticlockwise and negative when measured clockwise. The angle associated with a given point is not unique.

In some cases, we allow r to be negative. For instance, the point $P(2, 7\pi/6)$ can be reached by turning $7\pi/6$ radians anticlockwise from the initial ray and going forward 2 units, or we could turn $\pi/6$ radians anticlockwise and go backwards 2 units; corresponding to $P(-2, \pi/6)$.

Example 1: Find all the polar coordinates of the point $P(2, \frac{\pi}{6})$.

Polar Equations and Graphs: If we fix r at a constant value (not equal to zero), the point $P(r, \theta)$ will lie $|r|$ unites from the origin O. As θ varies over any interval of length 2π, P traces **a circle!**

If we fix θ at a constant value and let r vary between $-\infty$ and ∞, then the point $P(r, \theta)$ traces **a line!**

Example 2: A circle or line can have more than one polar equation.

Example 3: Equations of the form $r = a$ and $\theta = \theta_0$ can be combined to define regions, segments and rays. Graph the sets of points:

(a) $1 \leq r \leq 2$ and $0 \leq \theta \leq \dfrac{\pi}{2}$

(b) $-3 \leq r \leq 2$ and $\theta = \dfrac{\pi}{4}$

(c) $\dfrac{2\pi}{3} \leq \theta \leq \dfrac{5\pi}{6}$

Relating Polar and Cartesian Coordinates: When we use both polar and Cartesian coordinates in a plane, we place the two origins together and take the initial ray as the positive x-axis. The ray $\theta = \pi/2$, $r > 0$ becomes the positive y-axis. The two coordinate systems are then related by the following:

Example 4: Given the polar equation, find the Cartesian equivalent:

(a) $r\cos(\theta) = 2$

(b) $r^2 \cos(\theta) \sin(\theta) = 4$

(c) $r^2 \cos^2(\theta) - r^2 \sin^2(\theta) = 1$

(d) $r = 1 + 2r \cos(\theta)$

(e) $r = 1 - \cos(\theta)$

Example 5: Find a polar equation for the circle $x^2 + (y - 3)^2 = 9$.