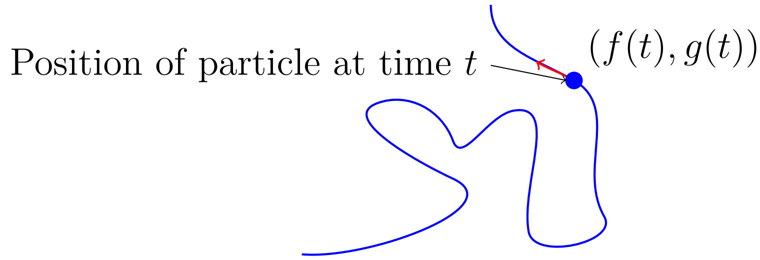
§ 11.1: Parametrisations of Plane Curves



Definitions: If x and y are given as functions $x = f(t), y = g(t)$ over an interval I of t-values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is a **parametric curve**. The equations are **parametric equations** for the curve.

The variable t is the **parameter** for the curve and its domain I is the **parameter interval**. If I is a closed interval, $a \leq t \leq b$, the **initial point** of the curve is the point $(f(a), g(a))$ and the **terminal point** of the curve is $(f(b), g(b))$.

When we give parametric equations and a parameter interval for a curve, we say that we have **parametrized** the curve. The equations and interval together constitute a **parametrization** of the curve. A given curve can be represented by different sets of parametric equations.

Example 1: Sketch the curve defined by the parametric equations $x = t^2, \quad y = t + 1, \quad -\infty < t < \infty$.

Example 2: Identify geometrically the curve in **Example 1** by eliminating the parameter t and obtaining an algebraic equation in x and y.

Example 3: Graph the parametric curves

$$\text{(a) } x = \cos(t), \quad y = \sin(t), \quad 0 \leq t \leq 2\pi,$$

$$\text{(b) } x = a\cos(t), \quad y = a\sin(t), \quad 0 \leq t \leq 2\pi, \quad a \in \mathbb{R}.$$

Example 4: The position $P(x, y)$ of a particle moving in the xy-plane is given by the equations and parameter interval

$$x = \sqrt{t}, \quad y = t, \quad t \geq 0.$$

Identify the path traced by the particle and describe the motion.

Example 5 - Natural Parametrisation: A parametrisation of the function $f(x) = x^2$ is given by

Example 6: Find a parametrisation for the line through the pt (a, b) having slope m.

Example 7: Sketch and identify the path traced by the point $P(x, y)$ if

$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}, \quad t > 0.$$