§ 8.4: Trigonometric Substitution

Common Trig. Substitutions:

Integral contains:	Substitution	Domain	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin(\theta)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$1 - \sin^2(\theta) = \cos^2(\theta)$
$\sqrt{a^2 + x^2}$	$x = a\tan(\theta)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	$1 + \tan^2(\theta) = \sec^2(\theta)$
$\sqrt{x^2 - a^2}$	$x = a\sec(\theta)$	$\left[0, \frac{\pi}{2}\right)$	$\sec^2(\theta) - 1 = \tan^2(\theta)$

Example 1: Evaluate $\displaystyle\int \frac{\sqrt{9 - x^2}}{x^2}\,dx$.

Example 2: Find $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$.

Example 3: Evaluate $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,dx$.

Example 4: Evaluate $\displaystyle\int \frac{dx}{\sqrt{25x^2-4}}, \qquad x > \frac{2}{5}$.

Example 5: Evaluate $\displaystyle\int \frac{x}{\sqrt{9 - x^2}}\, dx$.

Example 6: Evaluate $\displaystyle\int \frac{2\, dx}{x^3\sqrt{x^2 - 1}}, \qquad x > 1.$