§ 8.2: Integration by Parts

If $f(x)$ and $g(x)$ are differentiable functions of x, the **product rule** says that

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x).$$

Integrating both sides and rearranging gives us the **Integration by Parts** formula!

Let $u = f(x)$ and $v = g(x)$. Then, we obtain the **Integration by Parts Formula**:

Example 1: Find

$$\int x\cos(x)\,dx.$$

Example 2: Find

$$\int \ln x \, dx.$$

Example 3: Evaluate

$$\int x^2 e^x \, dx.$$

Example 4: Find

$$\int e^x \cos x \, dx.$$

Ex 4*: $\int e^x \sin x \, dx = \boxed{\dfrac{e^x(\sin x - \cos x)}{2} + C}$

Example 5 (Reduction formula): Obtain a formula that express the integral

$$\int \cos^n x \, dx$$

in terms of an integral of a lower power of $\cos x$.

For $n = 3$, we have $\displaystyle\int \cos^3 x \, dx = \frac{\cos^2 x \sin x}{3} + \frac{2}{3}\int \cos x \, dx = \frac{\cos^2 x \sin x}{3} + \frac{2}{3}\sin x + C.$

Ex 5* (Reduction formula): $\displaystyle\int \sin^n x \, dx = \boxed{\frac{-\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x \, dx}$

Integration by Parts for Definite Integrals:

$$\int_a^b f(x)g'(x)\,dx = f(x)g(x)\Big|_a^b - \int_a^b f'(x)g(x)\,dx.$$

Example 6 (Integration by Parts for Definite Integrals): Find the area of the region bounded by the curve $y = xe^{-x}$ and the x-axis from $x = 0$ to $x = 4$.