Math 142—Exam II

Instructor: Shaoyun Yi   Name: _____

⋆ **No calculators** are allowed during this exam.

⋆ You are required to show your work on each problem on this exam. The following rules apply:

• **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

• **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations or explanation will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

• **Indicate your final answer with a** $\boxed{\text{box}}$.

0. (Free 10 points) for taking the test. Enjoy!

1. [10 pts] Determine if the following statements are true or false: please **circle** true or false.

(a) True or False: If $\sum_{n=1}^{\infty} a_n$ is a series with $\lim_{n \to \infty} a_n = 0$, then the series converges.

(b) True or False: If a series converges, then that series converges absolutely.

(c) True or False: If $\sum_{n=1}^{\infty} a_n$ converges and $0 \le a_n \le b_n$ for all n, then $\sum_{n=1}^{\infty} b_n$ converges.

(d) True or False: If a series has a sequence of partial sums $\{S_n\}$ and $\lim_{n \to \infty} S_n = 1$, then the series converges to 1.

(e) True or False: The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/2}}$ converges.

2. [15 pts] Which series converge, and which diverge? Give reasons for your answers. If a series converges, find its sum.

(a) The geometric series $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{5}{4^n}$

(b) The telescoping series $\displaystyle\sum_{n=1}^{\infty}\Big(\ln(n)-\ln(n+1)\Big)$

3. [**15 pts**] Determine if the following series **converge** or **diverge**. You must explicitly state the name of any test you use.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 + n + 1}{n^4 + n^2 + 1}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

4. **[20 pts]** Determine if the following series **converge absolutely**, **converge conditionally**, or **diverge**. You must explicitly state the name of any test you use.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(-3)^n(n+1)}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$

5. [15 pts] Find the interval and radius of convergence for

$$\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n^2 + 5}}.$$

(Hint: Apply the Ratio Test and the Additional Discussions for the Endpoints.)

6. [15 pts] The series

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \cdots$$

converges to $\sin x$ for all x. Using this to answer the following questions.

(a) Find a series for $\cos x$.

(b) For what values of x should the series in part (a) converge?

Honor Statement: I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.
As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Printed Name: _____   Signature: _____

Problem	**0**	**1**	**2**	**3**	**4**	**5**	**6**	Total
Points	10	10	15	15	20	15	15	100
Score	10							