Review for Test 1

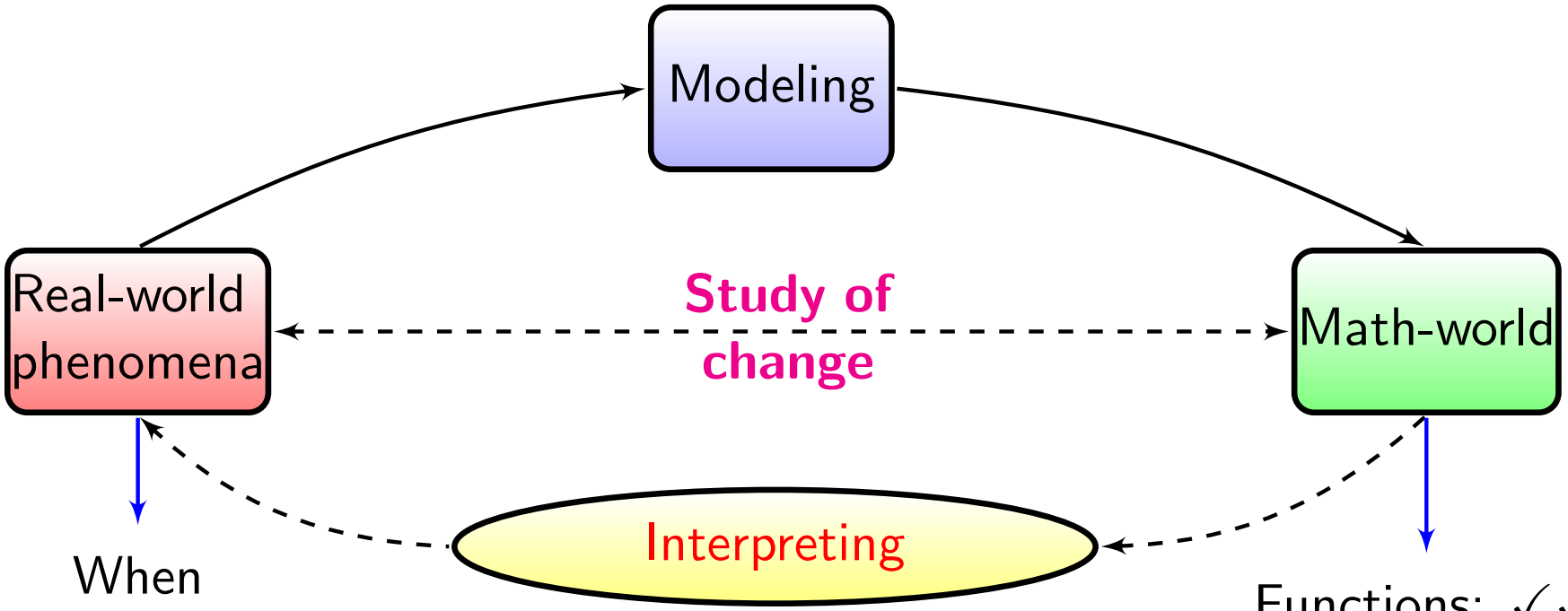
Shaoyun Yi

MATH 122

University of South Carolina

Fall 2020

Goal of This Semester



- Average R.O.C. $\overset{\Delta t \to 0}{\longrightarrow}$ R.O.C of $f(t)$ at $t = a$ (*Derivative* $f'(a)$).✓
- Accumulated Change $\overset{\Delta t \to 0}{\longrightarrow}$ Definite Integral $\int_a^b f(t)\,dt$ (**F.T.C.**).

Functions

Ask yourself: What is a function/domain (input)/range (output)?
Difference between closed/open intervals, i.e. "[" **v.s.** "(".
How to *interpret* your numerical answers correctly?
Horizontal intercepts (Zeros) **v.s.** Vertical intercept.
Behaviors of a function: increasing/decreasing/constant.
Concavity of a function: Concave up/down/neither (A line).

- **Linear:** $y = mx + b$ & $y - y_0 = m(x - x_0)$.
- **Exponential:** $P = P_0 \cdot a^t \overset{\text{base change}}{\longleftrightarrow} P = P_0 \cdot e^{kt}$
  - P_0 : Initial quantity (Vertical intercept)
  - r : Decimal representation of percent rate of change ($r = a - 1$)
  - k : *Continuous* growth/decay rate
  - Base Change: $a = e^k \iff \ln a = k$
  - $a > 1$ (so $r > 0$ and $k > 0$): Exponential growth
  - $1 > a > 0$ (so $r < 0$ and $k < 0$): Exponential decay
- **Natural Log.:** $y = \ln x \iff e^y = x \quad (x > 0)$
- **Power (Poly.):** $Q(x) = k \cdot x^p$ with the constant of proportionality k.
- *Composite Functions:* $f(g(x))$. Realize the inside function $u = g(x)$

Changes

1. Changes in y : $\Delta y = y_2 - y_1$. Unit: unit of y
2. Average Rate of Change: Unit: unit of y per unit of t

$$\frac{\Delta y}{\Delta t} = \frac{f(b) - f(a)}{b - a} = \text{Slope of Secant line (between } a \text{ and } b)$$

- **Linear:** Its slope m is the constant Average Rate of Change.
- $\Delta t \to 0$: (Instantaneous) Rate of Change (of f at a), i.e., $f'(a)$. And
  $f'(a) = \text{Slope of Tangent line at } A \text{ (or } x = a)$
  - Estimate $f'(a)$ by taking $\Delta t = 0.001$ in Average Rate of Change
  - Determine $f'(a)$ is $>$ / $<$ / $=$ 0 by reading the graph directly
  - Estimate $f'(a)$ given numerically (Usually, *Right-hand approximation*)
  - **First Derivative Test:** $f' > 0$ means $f \nearrow$ **v.s.** $f' < 0$ means $f \searrow$

3. Relative Change in P : $\frac{P_1 - P_0}{P_0}$ Unit: %

Applications

- Average Rate of Change: Average Velocity$= \dfrac{\text{change in distance}}{\text{change in time}}$
- **Linear:** Cost $C(q)$, Revenue $R(q)$, and Profit $\pi(q)$ Functions
  - $C(q) = C_{\text{fixed}} + C_{\text{variable}}$
  - $R(q) = p \cdot q$, where p is the price
  - $\pi(q) = R(q) - C(q)$
  - Break-even Point & Marginal Cost/Revenue/Profit (Slopes)
- **Exponential:** Know how to use "$a = 1 + r$" and "e^k" properly
  - Exponential growth: has a constant Doubling Time
  - Exponential decay: has a constant Half-Life
  - Compound Interest: $\begin{cases} \textit{annually} & P(t) = P_0 \cdot (1+r)^t \\ \textit{continuously} & P(t) = P_0 \cdot e^{rt} \end{cases}$
- **Natural Log.:** Solve equations using Natural Logarithms
- **Power:** Define a Polynomial (sum of power functions)
- *Composite Functions:* Produce New Functions from Old

How to Use Calculator

1. Graph a Function: See Slides of §1.3: # 4/9

   (i) $\boxed{y=}$: (ii) $\boxed{\text{window}}$: $X_{\min}/X_{\max}$ (iii) $\boxed{\text{zoom}}$: Choose "ZoomFit"

2. Plot a Table of Data: See Slides of §1.3: # 6/9

   (i) $\boxed{y=}$: (ii) $\boxed{\text{stat}}$: EDIT & 1: (iii) $\boxed{\text{zoom}}$: Choose "ZoomStat"

3. Find a Maximum: See Slides of §1.9: # 6/7

   (i) $\boxed{y=}$: (ii) $\boxed{\text{window}}$: $X_{\min}/X_{\max}$ (iii) $\boxed{\text{zoom}}$: Choose "ZoomFit"

   (iv) 2nd + $\boxed{\text{trace}}$: Choose "maximum" (v) Left/RightBound?/Guess?

4. Evaluate a Value–$Y_1(X)$: See Slides of §2.1: # 4/9

   (i) $\boxed{y=}$: (ii) $\boxed{\text{vars}}$: Y-VARS 1: 1: Y_1 (iii) Main Screen: $Y_1(X)$

Additional Suggestions

Review:

1. Your WileyPlus homework
2. Your class notes
3. Lecture recordings if necessary
4. Quizzes
5. Test 1 Review Problems with Solutions

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Good luck with the test!

Review for Test 2

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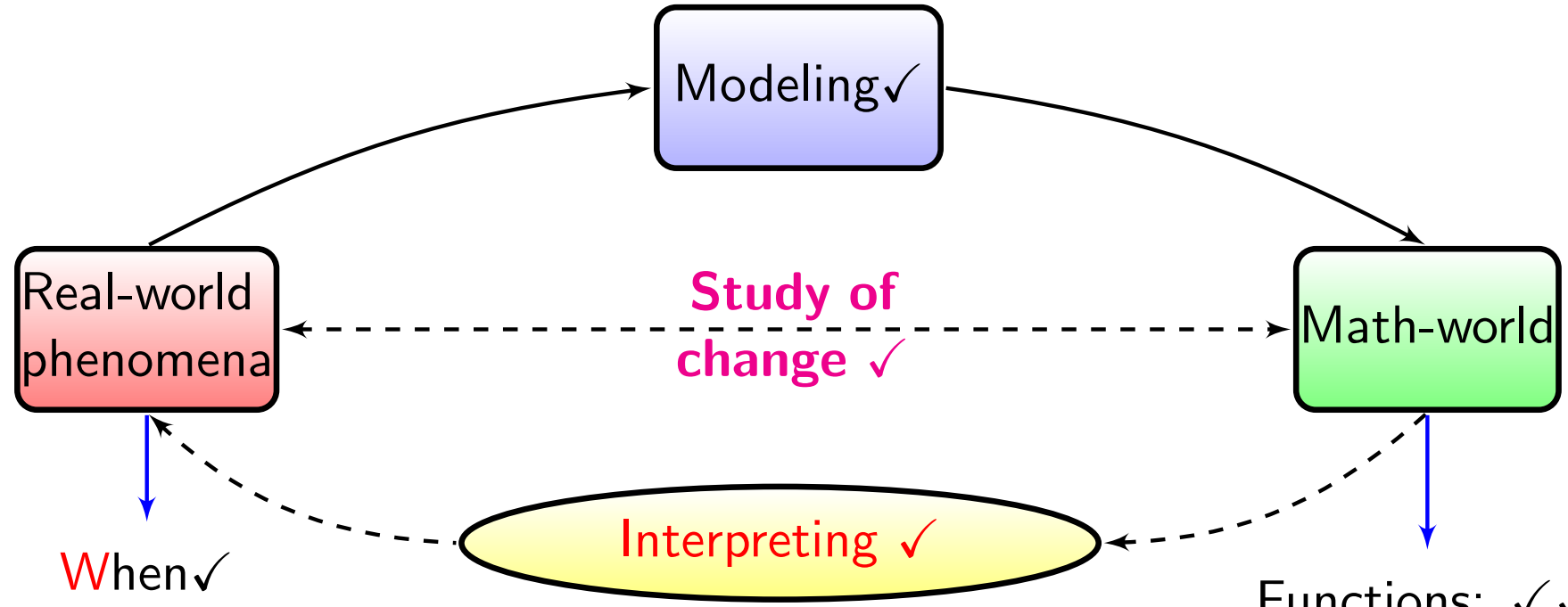

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(Instantaneous) Rate of Change/Derivative

1. Leibniz's Notation: $f'(x) = \dfrac{dy}{dx}$ and $f'(a) = \left.\dfrac{dy}{dx}\right|_{x=a}$
2. Interpretations: WWHHmU
3. Tangent Line Approximation: $f(x) \approx f(a) + f'(a) \cdot (x - a)$
4. Relative rate of change of y at $a = \dfrac{f'(a)}{f(a)}$ and Unit is $\underline{\%\text{ per unit of } t}$
5. **1st derivative test**: $f' > 0$ means f ↗ **v.s.** $f' < 0$ means f ↘
6. **2nd derivative test**: $f'' > 0$ means f 🙂 **v.s.** $f'' < 0$ means f 😕
7. All derivative rules: C, CM, S/D, Power/Exp/NLog, Chain, Prod./Quot.
   - Find Equation of tangent line
   - Find Second derivative
   - Find Marginal revenue/cost/profit
   - Find Specific values: *Examples in §3.3 and §3.4*
8. Local Maxima/Minima: Critical points ($f'(p) = 0$) & f' sign changes
   Method: if $f''(p) \neq 0$, **2nd derivative test**; otherwise, **1st derivative test**
9. Global Maxima/Minima: Compare critical values and endpoints values
   Application in real life: Find Maximum/Minimum profit
10. Inflection points (f concavity changes): $f''(p) = 0$ & f'' sign changes

Additional Suggestions

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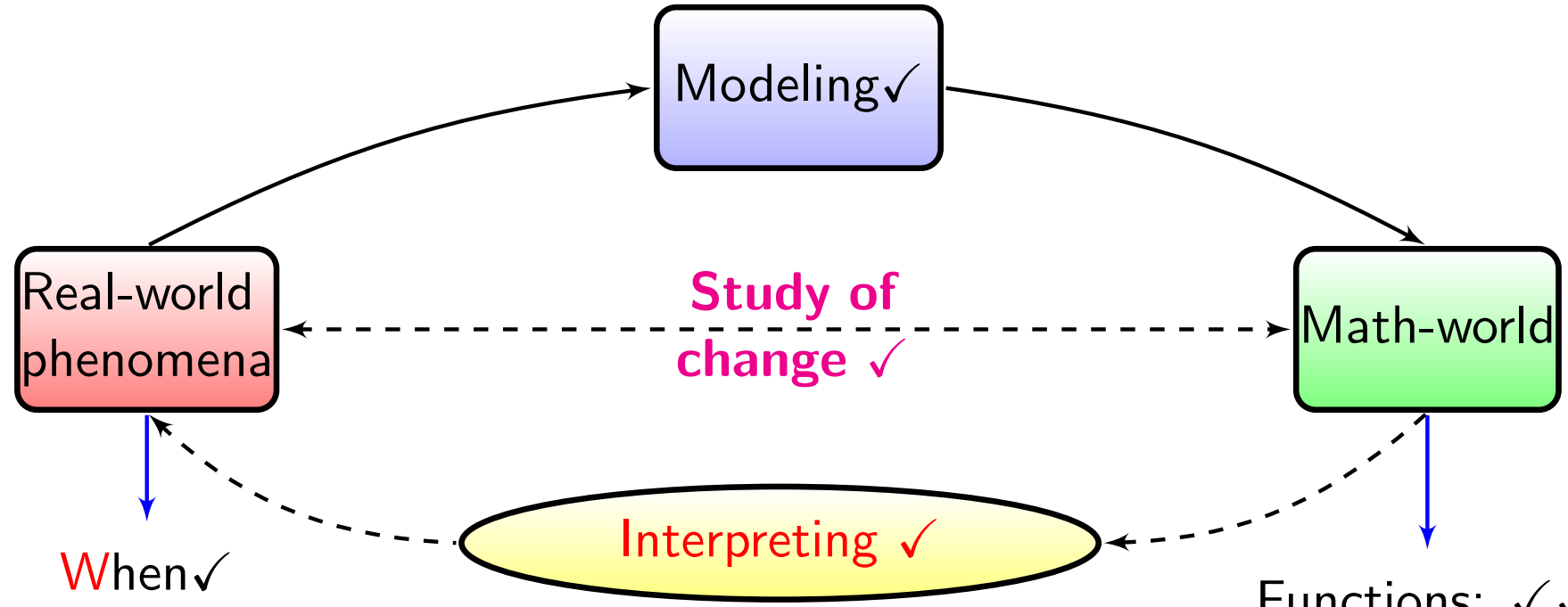

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Definite Integral

1. $\Delta t = \dfrac{b-a}{n}$:
   - **Left-hand sum** $= f(t_0)\Delta t + f(t_1)\Delta t + \cdots f(t_{n-1})\Delta t$
   - **Right-hand sum** $= f(t_1)\Delta t + f(t_2)\Delta t + \cdots f(t_n)\Delta t$
2. Definite integral: $\displaystyle\int_a^b f(t)\,dt = \lim_{n\to\infty} \big(\text{Left/Right-hand sum}\big)(\Delta t \to 0)$
   - *Use a calculator to evaluate a definite integral.* math & choose "9."
   - Geometric side: If $f(t) > 0$, then $\displaystyle\int_a^b f(t)\,dt =$ area under graph of f.

     If $f(t) < 0$, then $\displaystyle\int_a^b f(t)\,dt = -$ area between a and b

     Area between two curves.
   - Estimate a definite integral numerically/graphically: **Left-/Right- sum**
   - Interpretation: Unit for $\int_a^b f(t)\,dt =$ product of unit of f and unit of t
   - The Fundamental Theorem of Calculus: $\displaystyle\int_a^b F'(x)\,dx = F(b) - F(a)$.

     e.g. Total cost $C(b)$ of producing b units: $C(b) = C(0) + \displaystyle\int_0^b C'(q)\,dq$.

Indefinite Integral & Use FTC to evaluate definite integrals

1. Antiderivative: If $F'(x) = f(x)$, then $F(x)$ is an *antiderivative* of $f(x)$. Analyzing antiderivative graphically (FDT) & numerically (Calculator)

2. The Indefinite Integral of $f(x)$: $\int f(x)\,dx = F(x) + C$
   - The *family* of antiderivatives of $f(x)$;
   - **Formulas for Antiderivatives:** $f(x) = k$; $x^n\,(n \neq -1)$; $\frac{1}{x}$; e^x; e^{kx};
     Properties of Antiderivatives: Sums and Constant Multiples
   - Integration by Substitution (u-substitution): $u = u(x)$ & $du = u'(x)\,dx$

3. Using the Fundamental Theorem to evaluate Definite Integrals:
   (i) Find an antiderivative $F(x)$, i.e., Calculate $\int f(x)\,dx = F(x) + C$
   (ii) Evaluate the definite integral: $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$
   (iii) A warm tip: *You can use a calculator to double-check your answers!*
   (iv) Applications: • Find the area; • Evaluate definite integrals with u-sub.

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